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EXTREME SEA WATER TEMPERATURES AND DENSITIES ALONG THE NORTH ATLANTIC STATES DURING SUMMER 1955

By

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ABSTRACT

During the summer of 1955, record high sea water temperatures were observed in harbors and coastal waters of the North Atlantic States. Record air temperatures also occurred during this period. It is concluded that high sea water temperatures were due in large part to continued high air temperatures. The record low densities (or salinities) during the same summer corresponded with the periods of hurricanes Connie and Diane, hence the high runoff following these storms was doubtless responsible. The lag between rainfall and noticeable dilution and the delay in return to normalcy depend upon several factors including the amount of rain, type of estuary, and size of drainage basin.

The summer of 1955 was notable for the record high sea water temperatures and the extremely low densities observed in harbor and coastal waters of the North Atlantic States.

For many years and at many places along the Atlantic Coast, the Coast and Geodetic Survey has observed daily the temperature and density of the sea and has determined average monthly conditions from these records. Densities were observed by hydrometer, and the resulting data are given as density at 15° C (59° F) referred to the density of pure water at 4° C as unity. Temperatures are shown in degrees Fahrenheit. Air temperature and rainfall data were obtained from published reports of the U. S. Weather Bureau.

TABLE I. Highest Monthly Mean Water Temperatures (°F)

	1955 High		Previous High		Next Previous High		Number of monthly means that equaled or exceeded those of 1955	
	Temp.	Month	Temp.	Month, Year	Temp.	Month, Year		Series
Eastport, Me.	52.3	Aug.	55.2	Sept. '49	54.8	Sept. '47	21	1930–55
Bar Harbor, Me.	59.2	Aug.	59.1	Aug. '51	58.6	July '51	0	1948–55
Portsmouth, N. H.	64.6	Aug.	64.0	Aug. '47	63.5	Aug. '51	0	1944–55
Boston, Mass.	70.7	Aug.	69.5	July '49	69.2	Aug. '49	0	1921–55
Woods Hole, Mass.	73.2	Aug.	73.0	Aug. '52	72.5	Aug. '47 Aug. '49	0	1944–55
New London, Conn.	76.1	Aug.	74.1	Aug. '52	74.0	July '55	0	1947–55
Montauk, L. I., N. Y.	72.6	Aug.	71.8	Aug. '49	71.2	July '49 Aug. '52	0	1948–55
Willets Point, N. Y.	75.1	Aug.	74.9	Aug. '49	74.1	Aug. '41	0	1932–55
New York (The Battery), N. Y.	76.5	Aug.	77.2	Aug. '39	75.7	Aug. '49	1	1927–55
Sandy Hook, N. J.	78.0	Aug.	77.6	July '52 July '55	77.5	July '87	0	1887–93 & 1944–55
Atlantic City, N. J.	75.1	Aug.	76.1	Aug. '39	75.4	Aug. '49	2	1912–20 & 1923–55
Breakwater Harbor, Del.	77.5	Aug.	75.9	Aug. '53	75.8	July '48	0	1919–22, 1947–49 & 1952–55
Philadelphia, Pa.	84.1	July	82.4	July '34	82.0	Aug. '39	0	1922–46 & 1954–55
Baltimore, Md.	82.9	July	82.9	July '35	82.8	July '34	1	1914–55
Annapolis, Md.	82.8	July	82.3	July '53	82.2	July '52	0	1947–55
Washington, D. C.	85.3	July	85.1	July '44 Aug. '47	84.9	July '52 July '53	0	1944–55
Solomons, Md.	82.4	July	82.2	Aug. '38	81.9	July '53	0	1938–55
Richmond, Va.	84.5	July	84.8	July '53	84.2	July '52	1	1947–55
Gloucester Pt., Va.	83.4	Aug.	83.3	July '55	82.7	Aug. '51 July '52	0	1950–55

In examining the weather records for the possible cause of these extremes, we find that warm air over much of the United States, including the northeastern section of the country, set new records or approached old ones for highest average monthly air temperatures. Moreover, during August, heavy rains resulting from hurricanes Connie and Diane flooded drainage systems in the New England and Middle Atlantic States and greatly increased the fresh water discharge into the usually saline bays and estuaries.

A tabulation of the highest monthly mean water temperature for each station along the North Atlantic for the year 1955 is given in Table I. Also given are the previous two highest values and the dates when they occurred. The last columns show the total number of times the highest 1955 mean was equaled or exceeded during previous years as well as the series duration for each station. From this tabulation, covering stations from Maine to Virginia, it is seen that 1955 monthly means equaled or exceeded the records set during previous years at 15 of the 19 stations for which data are available.

Let us examine the air temperatures for several stations in this area for which Weather Bureau data are available. These values together with sea water temperatures are given in Table II. This table reveals that sea water temperatures during July and August 1955 averaged 2.5° F above normal and air temperatures were 3.9° F above normal.

TABLE II. Mean Air and Water Temperatures for July-August (°F)

		Air (1921–1950)			*Water**		
Station		*1955*	*Av.*	*Diff.*	*1955*	*Av.*	*Diff.*
Boston	July	77.2	72.2	+5.0	68.7	64.5	+4.2
	Aug.	74.5	71.5	+3.0	70.7	65.6	+5.1
New York	July	79.7	74.6	+5.1	73.1	71.6	+1.5
	Aug.	77.2	73.2	+4.0	76.5	73.3	+3.2
Atlantic City	July	76.0	73.6	+2.4	70.2	69.5	+0.7
	Aug.	76.3	73.1	+3.2	75.1	71.7	+3.4
Philadelphia	July	81.8	77.2	+4.6	84.1	78.6	+5.5
	Aug.	78.5	75.2	+3.3	79.7	78.4	+1.3
Baltimore	July	83.5	78.5	+5.0	82.9	79.6	+3.3
	Aug.	79.2	76.4	+2.8	81.2	79.6	+1.6
Washington	July	82.9	77.8	+5.1	85.3	83.6	+1.7
	Aug.	79.1	75.8	+3.3	80.7	82.1	−1.4
		Av. Difference +3.9			Av. Difference +2.5		

* For length of series, see Table I.

Since air temperatures appear to have such an important effect on sea water temperatures, let us study the meteorological conditions for the stations shown in Table II. The Weather Bureau reports that the July mean temperature for Boston was within 0.3° of the all-time record for July (77.5°, set in 1952) and that August 1955 was the third warmest August since 1871. At New York the average monthly air temperature for July established a new record, the previous one being 78.9° in 1952; August was the second month in a row to set a heat record. The July mean at Philadelphia also was a new record for that month. The average monthly temperature for July in Baltimore broke the old record of 81.5° set in 1872. Washington continued this record breaking precedent by exceeding the highest July average temperature of 81.4° set in 1949. Thus, when it is seen how many air temperature records were set, it is easy to understand why the water temperatures also were high.

Whereas the extreme sea water temperatures can be explained in large part through a correlation with air temperatures, we need to study the precipitation data to explain the extremely low sea water densities. During August 1955 two tropical hurricanes, Connie and Diane, hit the northeastern section of the country, bringing heavy rains generally along the entire coast from North Carolina northward and eastward through southern New England. Sea water density data, and precipitation data for the same stations where available, are shown in Table III. Stations north of Boston were little affected and are not included in this table.

The daily densities of the monthly records fell naturally into the groupings shown in Table III. It is seen that they correspond with periods of light and heavy rainfall. Densities at stations located in harbors and estuaries were affected by the increased runoff. Up-river stations such as Washington and Richmond showed little effect, for their waters are practically fresh at all times. Normally saline coastal stations such as Woods Hole, Montauk, and Atlantic City showed no unusual density fluctuations.

Hurricane Connie was active August 11–13 in eastern Virginia, Maryland, Delaware and Pennsylvania. Some rain fell in New England, but it was minor when compared with that produced by Hurricane Diane, which began moving inland in the vicinity of Wilmington, North Carolina early on the 17th of August. This storm continued on a more or less northerly path until it approached the Mason-Dixon Line, where it turned eastward on August 19 and passed almost directly over Philadelphia; from there it continued northeastward along the southern shores of New England. Prior to these two storms there had been a long dry spell in the New England

TABLE III. Rainfall and Sea Water Density Data for August 1955

	Rainfall		*Density*			
	Dates	*Total (inches)*	*Dates*	*Average*	*1955 Low*	*Previous Low*
Boston, Mass.	1–10	0.28	1–10	1.0222		
	11–13	1.47	11–15	1.0209		
	14–16	0.00	17	1.0226		
	17–19	12.47	18–31	1.0102		
	20–31	2.87	19		1.0029	1.0136
Woods Hole, Mass.	—	—	1–11	1.0236		
	—	—	12	1.0227		
	—	—	13–17	1.0235		
	—	—	18–31	1.0232		
New London, Conn.	—	—	1–13	1.0205		
	—	—	14–31	1.0065		
	—	—	28		1.0028	1.0083
Montauk, L. I., N. Y.	—	—	1–22	1.0224		
	—	—	23	1.0188		
	—	—	24–31	1.0220		
Willets Point, N. Y.	1–10	1.29	1–11	1.0195		
	11–14	12.50	12–14	1.0160		
	15–17	0.00	15–31	1.0179		
	18–19	1.73	12		1.0157	1.0164
	20–31	0.53				
New York (The Battery) N. Y.	1–6	0.00	1–12	1.0186		
	7	2.20	15–19	1.0162		
	8–10	0.07	22–26	1.0097		
	11–14	9.60	29–31	1.0141		
	15–17	0.00	23		1.0081	1.0095
	18–19	1.62				
	20–31	0.33				
Sandy Hook, N. J.	—	—	1–13	1.0207		
	—	—	15–31	1.0145		
	—	—	22		1.0103	1.0127
Atlantic City, N. J.	1–7	0.00	1–20	1.0235		
	8	1.36	22–29	1.0229		
	9–11	0.06				
	12–13	2.26				
	14–30	0.56				
	31	1.66				
Breakwater Harbor, Del.	—	—	1–19	1.0229		
	—	—	21–31	1.0191		
	—	—	29		1.0162	1.0199

TABLE III.—*Continued*

	Rainfall		Density			
	Dates	Total (inches)	Dates	Average	1955 Low	Previous Low
Philadelphia, Pa.	1–11	0.95	1–12	0.9996		
	12–13	3.90	15–19	0.9991		
	14–16	0.02	22–26	0.9994		
	17–19	3.51	29–31	0.9991		
	20–31	0.55				
Baltimore, Md.	1–5	0.01	1–12	1.0071		
	6–7	2.08	15–19	1.0034		
	8–11	0.16	22–31	1.0051		
	12–14	9.07				
	15–16	0.00				
	17–18	3.58				
	19–31	2.79				
Annapolis, Md.	—	—	1–12	1.0092		
	—	—	13–24	1.0085		
	—	—	25–30	1.0069		
Washington, D. C.	—	—	1–31	0.9992		
Solomons, Md.	—	—	1–13	1.0115		
	—	—	14–19	1.0106		
	—	—	22–31	1.0092		
Richmond, Va.	—	—	1–31	0.9993		
Gloucester Point, Va.	—	—	1–11	1.0169		
	—	—	13–18	1.0148		
	—	—	19–31	1.0090		

and Middle Atlantic States, making the land very dry. Rains from Connie, therefore, were absorbed by the soil. When Diane's rains hit the area less than one week later, the ground was too soaked to absorb the additional water. The result was flooding of all streams. This high runoff, continuing for several days, sharply reduced the density of sea water in harbors such as Boston, New London, and New York.

Figs. 1 and 2 show graphically the daily record of density and rainfall for Boston Harbor and New York Harbor. Where daily density observations are missing, dashed lines have been used to connect the observed portions of the curve. It will be seen that there was a lack of rain in early August so that densities in the harbors were high. Then the rains came and the density was lowered due to runoff. Rains from Connie were heavier at New York whereas Diane dumped more water on the Boston area. From the graph it is seen that at New York there was considerable lag between the rainfall and the resulting decrease in harbor density, whereas a

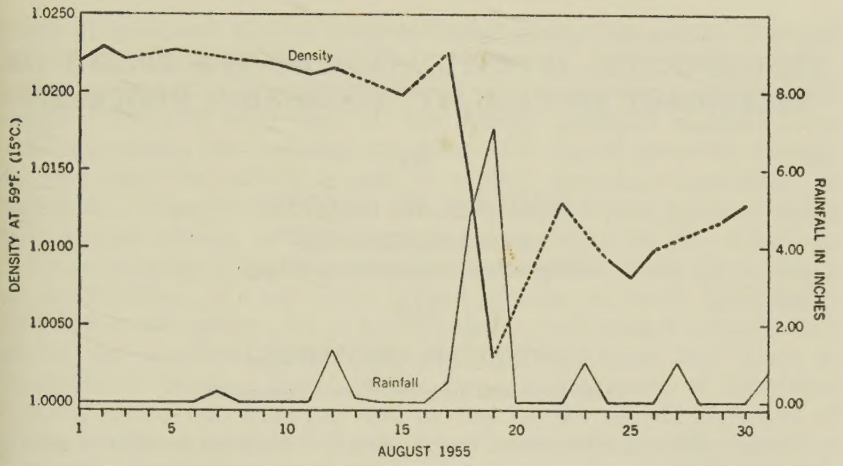

Figure 1. Daily sea water density and rainfall for Boston; August 1955.

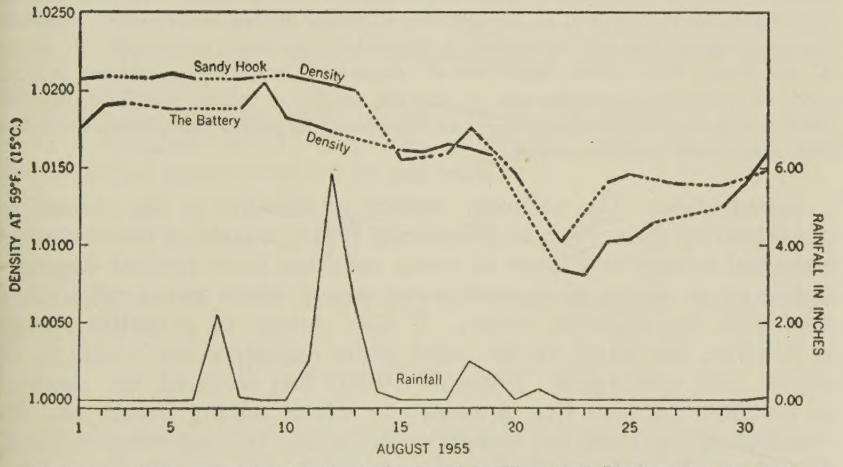

Figure 2. Daily sea water density and rainfall for New York Harbor; August 1955.

Boston there was practically no lag at all. This time difference reflects the different types and sizes of drainage basins. Waters in New York Harbor are the accumulation of many rivers emptying into the Hudson River over a drainage area of many square miles. At Boston the watershed is small and runoff is emptied almost immediately into the Harbor, resulting in quickly lowered density. There are similar lags between the end of the rain and the beginning of increasing density. Though the increase in density is rather rapid for a while, it shows some delay, possibly due to continuing runoff and to mixing with the saline waters entering from the sea.

EXPERIMENTAL INVESTIGATION OF THE EFFECT OF "SECONDARY INSTABILITY" ON MIXING PROCESSES[1]

By

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Abstract. Since specific volume in the ocean is a nonlinear function of salinity and internal energy, mixing results in a change in the total volume. There is a contribution to the change in potential energy due to mixing, which depends on this change in volume and on the pressure at which mixing takes place. Fofonoff has made use of this fact in defining a "secondary stability" parameter. It might be speculated that, under conditions of (secondary) instability, turbulent mixing could be generated spontaneously or that the quality of mixing would be different from that under stable conditions. An experiment to detect this phenomenon was performed, with negative results.

Introduction. The ordinary concept of stability in the oceans, as developed by Hesselberg and Sverdrup (1914), considers the change of potential energy in a layer of water resulting from vertical displacement, up or down, of a small water parcel which is not allowed to mix with the adjacent water. If this change of potential energy is positive, the water at the point under consideration is said to be *stable*, and vice versa. Fofonoff (1956) has widened the concept of stability by including processes in which mixing occurs and by considering the potential energy of the entire vertical water column. The term pdv is thus introduced as an additional contribution to the potential energy change. This term results from the fact that specific volume is a nonlinear function of salinity and internal energy and is usually negative; thus an unstable configuration according to Hesselberg and Sverdrup's criterion will usually be unstable according to Fofonoff's criterion. Hesselberg and Sverdrup's stability is referred to as "primary" stability, Fofonoff's as "secondary." The experiment described in this note was an attempt to observe any possible dramatic effects associated with secondary instability (such as the

[1] Contribution from the Scripps Institution of Oceanography, New Series No. 989.

onset of turbulent motion from an initial motionless state); Fofonoff has made no such speculation.

Let us consider a configuration in which density increases with depth. Fofonoff finds that if the density gradient exceeds some *critical gradient,* then mixing can occur with loss of potential energy; in that case the system is said to exhibit secondary instability at this level. Therefore a system consisting of two homogeneous layers will almost always exhibit secondary instability at the interface.

To get a rough idea of the amount of mixing that would be necessary at the interface of a two-layer system in order to reach (secondary) stability in the system, let us assume that a fluid mass m immediately above the interface mixes thoroughly with an equal fluid mass m immediately below. Let A be the cross-sectional area of the water column under consideration and α_1 and α_2 the specific volumes of water in the upper and lower layers, respectively. For primary stability, we must have $\alpha_1 > \alpha_2$. After mixing there will be a homogeneous layer of water of mass $2m$ between the two original layers having a specific volume, say, of α. This layer is assumed thin enough so that inhomogeneity due to compressibility and pressure gradient may be neglected.

Before mixing, the center of gravity of the water to be mixed is at a distance $(m/4A)\ (3\alpha_2 + \alpha_1)$ above its lower boundary, and the potential energy relative to this level is

$$\frac{m^2 g}{2A}\ (3\alpha_2 + \alpha_1).$$

After mixing, the center of gravity will be at $\alpha m/A$ and the potential energy will be

$$\frac{2m^2 \alpha g}{A} + mp\ (2\alpha - \alpha_1 - \alpha_2),$$

where the second term represents the pdv addition to the potential energy. Equating the potential energies before and after mixing to find the amount of water that must mix in order to reach (secondary) stability, we obtain

$$\frac{m}{A} = \frac{2p}{g} \cdot \frac{\alpha_1 + \alpha_2 - 2\alpha}{4\alpha - 3\alpha_2 - \alpha_1}, \tag{1}$$

and it is evident that if the specific volume is governed by a linear law, no water can mix without addition of potential energy.

The Experiments. A two-layer system, consisting of pure water above and a 17% NaCl solution colored with red vegetable dye

below, was put under 1000 psi pressure for 18 hours. The tank used was a rectangular aquarium 27 cm long, 17 cm wide, and 19 cm deep. In order to apply the relationship (1) to this configuration we shall neglect the change of specific volume resulting from compressibility and from conversion of potential to internal energy (which is unimportant at this pressure anyway). (Note that compressibility would play no role whatsoever if the specific volumes α, α_1 and α_2 were all affected proportionately by the pressure.) We shall therefore use the following values of specific volume for NaCl solutions at atmospheric pressure and at 20° C:

%NaCl	Specific Volume
0	1.0018
8.5	.9438
17	.8897

Taking $p = 1000\ psi = 68 \times 10^6$ dynes cm^{-2} along with the above values, we obtain $m/A = 6 \times 10^3$ gm cm^{-2} from relationship (1), which is equivalent to a thickness of 60 m. Even at atmospheric pressure, the thickness over which mixing should occur without need of additional potential energy is of the order of a meter. The motive in going to 1000 psi was to increase the secondary instability and the likelihood of a spectacular result.

At the end of 18 hours there was no visual evidence of turbulent mixing, but of course the sharpness at the interface had become considerably reduced as a result of molecular diffusion. A stirring apparatus consisting of ¼″ wire mesh mounted horizontally in the upper layer was then turned on, the mesh executing reciprocal up and down motion;[2] no qualitative difference could be detected visually in the character of mixing at 1000 psi as compared with that at atmospheric pressure.

Even if secondary instability does not induce turbulent mixing from an initial motionless state, perhaps mixing by molecular diffusion is speeded up. The first experiment would have given no indication of this, since the dye, which was used as a visual tracer, contributes negligibly to changes in specific volume and could diffuse independently of the NaCl. Therefore, a second experiment was carried out with fresh water and a concentrated solution of $CuCl_2$ (just under saturation at room temperature). This solution itself is bright green and no dye is needed as a tracer. As before, the two layer system was submitted to 1000 psi for 18 hours, at the end of which time there was no striking

[2] The junior author has used similar apparatus in a mixing experiment at atmospheric pressure.

evidence of a qualitative difference in the diffusion rate under pressure as compared with that under atmospheric pressure. It must be emphasized that we were not set up for a quantitative experiment on diffusion rates; by the methods at our disposal we could have detected only a gross difference.

Conclusions. A simple experiment, carried out in an attempt to detect a possible effect of secondary instability, gave a negative result; this, of course, does not prove that there are no such effects. It seems probable, however, that the principal result of secondary instability, if such exists at all, is a small change in the diffusion rate in unstable configurations. In the case of primary instability, it is quite evident that the potential energy lost to the system as a result of a small perturbation tends to increase the perturbation. For secondary stability this loss of energy may go somewhere else (directly into heat, for example). In any case, energy considerations in the initial and final states alone are inadequate for predicting the outcome of such an experiment. Henry Stommel (personal communication) has suggested the possible existence of a criterion for the onset of convection in this stability problem that is similar to Rayleigh's convection criterion.

Acknowledgments. A great deal of insight into the problem was obtained through many talks with Dr. Carl Eckart of the Scripps Institution of Oceanography and through correspondence with Dr. N. P. Fofonoff, who at the time was with the National Institute of Oceanography in England. The authors thank the U. S. Navy Electronics Laboratory of San Diego, especially Mr. Clarence Milligan of that establishment, for use of its high pressure tank. Instrumentation was provided through contract with the U. S. Navy Bureau of Ships. This note represents a result of research conducted by the California Cooperative Oceanic Fisheries Investigations and the Inter-American Tropical Tuna Commission.

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THE BATHYSTROPHIC STORM TIDE[1]

By

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ABSTRACT

The tides on a regular open coast change rapidly as a storm approaches, and the time history of this change is important in computing dangerously high tides in bays; in addition, this change influences the maximum wave height offshore. Simple physical assumptions are used (and justified) to define a "bathystrophic" flow, and it is shown mathematically that this flow is of great importance in the computation of storm tides. Using only the traditionally accepted formulae and constants for wind stress and bottom friction, examples of the time change of the tide have been calculated. These results agree with observations as well as can be expected. Deviations from observations may be due to an edge wave.

INTRODUCTION

The storm tide is the primary cause of death and property damage in a hurricane, and as such, the study and forecast of storm tides are among the most important practical problems in meteorology and oceanography. We submit, here, a simple theory of coastal storm tides and the resulting forecast method.

The storm tide is a function of time and of at least two space dimensions. Presumably the tide can be computed on an electronic computer by using the complete set of two-dimensional equations of motion and continuity.[2] Freeman and Baer (1957) have described a "method of wave derivatives" which is closely related to the physics of the problem and which can be carried out graphically. But these methods of computation are expensive and difficult to use as presently developed, therefore a simple approach to storm tides and a facile forecast method are called for.

[1] The research and preparation of this paper was supported entirely by Gulf Consultants as part of a program for attracting outside support for both theoretical and practical studies of storm tides and water levels.

[2] We might add this result of the present work: When we do the full problem on a computer we expect to compute deviations from the bathystrophic storm tide in addition to the bathystrophic storm tide itself.

Consideration of the significance of the various terms in the equations of motion and continuity and of certain physical assumptions leads to a quasi-one-dimensional concept of the large-scale and long-term changes in the coastal storm tide. This coastal storm tide makes a significant contribution to the storm tide in any basin that is connected with the sea on a continental shelf.

DISCUSSION

Assumption 1.[3] On an open coastline with regular depth contours there is no sustained transport toward shore across the contours. If there were, much larger land areas would be inundated than are observed. For example, a 20 knot wind blowing on shore for 10 hours, if uncompensated, would cover an area up to 60 miles inland with a flood six feet deep. This is computed from the formulae[4]

$$\frac{\partial F_x}{\partial t} = \frac{\tau_x}{\rho} = 12 \times 10^{-4} \text{ knot}^2 \text{ for a 20 kt wind;}$$

$$\int_0^{10} F_x \, dt = 6 \times 10^{-2} \text{ mi.}^2 \text{ per mile of coast}$$

$$= 6 \text{ ft. deep} \times 60 \text{ mi. inland.}$$

The equivalent also holds true for offshore flow. Thus the only sustained currents toward shore are insignificant.

This postulates, then, that a compensating sustained current does exist parallel to the depth contours, and we define it as the "bathystrophic" current. The concept is somewhat analogous to that of the "geostrophic" wind in the atmosphere.

Assumption 2. Divergence of the velocity field does not bring about significant changes in the height of the water surface. Consider a mound of water which is superimposed on a level sea with a shoaling bottom; the mound is leveled by the action of gravity. In this case the edge of the mound expands at speeds of the same order of magnitude as the wave speed. Stationary disturbances, maintained by divergence in the velocity field, must be in currents with speeds of the order of magnitude of the wave speed. The greatest current which could act to build up the water mound (from six fathoms depth out to sea) is less than five knots. All of the wave speeds at which the mound edges are diverging are greater than 14 knots. Thus the dissipation of such a mound proceeds at a faster rate than its potential building up.

[3] This first assumption of our development is further supported in the APPENDIX.

[4] See list of symbols for definitions.

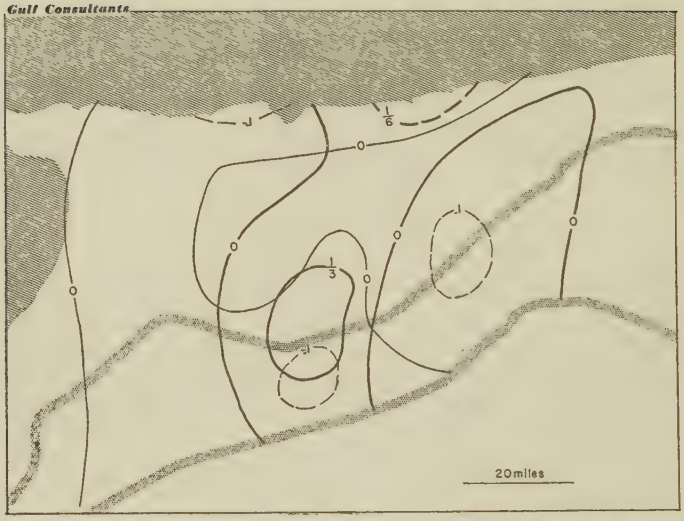

Figure 1. The heavy lines give the deviation of the wave speed that would build up if a divergence field acted for one hour without compensation; 1/3 is equivalent to about 2 feet of tide. The light lines give the deviation of the wave speed after the two dimensional equations of motion and continuity have been acting for ten minutes.

An example of the dissipation of the water heights built up by divergence is repeated here from Freeman and Baer (1956); see Fig. 1. A divergent current field is given and it is assumed that it works (uncompensated) for an hour to build up a height deviation in the water of about two feet (⅓ knot in diagram). This deviation is dissipated in 10 minutes to about 0.6 foot; further, the full computation shows that the current field becomes adjusted to discourage further build-up. In essence, we have assumed that a small amount of cross-contour transport develops to compensate for the divergence in the bathystrophic current; thus, the continuity equation need not be considered in our development.

Assumption 3. As we move in the y-direction the height changes are insignificant. A transport along the height contours can become very large without changing the height of the tide. (A transport normal to the depth contours causes changes in the height of the tide.) There is nothing equivalent to a barrier which allows water to pile up and give significant height differences in the y-direction.

Assumption 4. We assume that space derivatives of the current speed can be neglected with respect to the Coriolis parameter. We are

discussing space scales of the order of 25 to 50 miles, and the maximum current we can expect is 5 knots. If a change in speed of 5 knots in 25 miles occurred, as an extreme example, we would have space derivatives of the current that are of the same order as the Coriolis parameter. In the normal situation, we assume that there is a change of the order of 1 knot in 50 miles; thus,

$$\frac{\partial v}{\partial y} = 0.02 \ll 0.25 = f, \text{ the Coriolis parameter.}$$

Bathystrophic Current and Storm Tides. As a consequence of these four assumptions, we have a quasi-one-dimensional (time-dependent) problem, and the solution can be found at each point for which the time history of the wind is known. The solution consists of determining the flux parallel to the contours as well as the derivative normal to the contours of height of the storm tide. In order to find the tide, it is necessary to integrate the derivative from deep water to the point for which the tide is desired. In our first computations we neglected the bottom friction term. This led to the results shown in Figs. 3 and 4 and to the logical absurdity that a sustained wind would give an ever-increasing storm tide. Thus it is shown that bottom friction must be included to allow computation of the falling storm tide.

Mathematical Development. We study the flow in a curved, orthogonal, right-hand coordinate system with the y-direction parallel to, and the x-direction normal to, the depth contours. The x-direction is positive toward shore.

The discussion will be facilitated by using the following symbols:

F_y = flux parallel to depth contours $= \int_{-h}^{\eta} u \, dz$

F_x = flux normal to depth contours $= \int_{-h}^{\eta} v \, dz$

u = current speed in x-direction
v = current speed in y-direction
t = time
z = vertical coordinate
g = acceleration of gravity
h = mean low water depth
η = deviation from mean low water depth
τ_x = wind stress in x-direction
τ_y = wind stress in y-direction

ρ = density of water
f = Coriolis parameter
k = bottom friction coefficient
τ_b = bottom stress

The assumptions discussed above may be expressed:

$$|F_x| \ll |F_y| \tag{1}$$

$$\left|\frac{\partial F_y}{\partial y}\right| \ll \left|\frac{\partial \eta}{\partial t}\right| \tag{2}$$

$$\left|gh\frac{\partial \eta}{\partial y}\right| \ll \left|\frac{\tau_y}{\rho}\right| \tag{3}$$

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y} \ll f \ . \tag{4}$$

Using Assumption 4 and integrating the equations of motion for fluid flow in the z-direction, we obtain the following:

$$\frac{\partial F_x}{\partial t} = -gh\frac{\partial \eta}{\partial x} + f F_y + \frac{\tau_x}{\rho} - \frac{\tau_{bx}}{\rho} \tag{5}$$

$$\frac{\partial F_y}{\partial t} = -gh\frac{\partial \eta}{\partial y} - f F_x + \frac{\tau_y}{\rho} - \frac{\tau_{by}}{\rho}. \tag{6}$$

We now apply Assumptions 1 and 3 to obtain

$$gh\frac{\partial \eta}{\partial x} = f F_y + \frac{\tau_x}{\rho} - \frac{\tau_{bx}}{\rho} \tag{7}$$

$$\frac{\partial F_y}{\partial t} = \frac{\tau_y}{\rho} - \frac{\tau_{by}}{\rho} \ . \tag{8}$$

We use Manning's Formula as given by Linsley, *et al.* (1949) to find the bottom friction term. This gives us

$$\frac{\tau_{by}}{\rho} = \frac{k}{h^{7/3}} F_y^2. \tag{9}$$

and

$$\frac{\tau_{bx}}{\rho} = 0 \tag{10}$$

by virtue of Assumption 1. Thus Equations (8) and (7) become

$$\frac{\partial F_y}{\partial t} = \frac{\tau_y}{\rho} - \frac{k}{h^{7/3}} F_y^2 \tag{11}$$

$$gh\frac{\partial \eta}{\partial x} = f F_y + \frac{\tau_x}{\rho}. \tag{12}$$

Equations (11) and (12) are expressed only in terms of the two unknowns, η and F_y. These are the prediction equations for the bathystrophic storm tide. At a given time the value of F_y is found by integrating (11) from the time when a significant wind began. Then η is found at any point by integrating (12) from deep water where η is assumed to be zero.

Note that we have not used the continuity equation. By virtue of Asumptions 1 and 2 we find that $\partial\eta/\partial t$, as given by the continuity equation, involves terms we neglect $\left(\text{i. e., } \frac{\partial F_x}{\partial x} \text{ and } \frac{\partial F_y}{\partial y}\right)$. Thus we may find η from the quasi-steady state conditions without using the continuity equation.

Some Samples of the Bathystrophic Storm Tide. Equations (11) and (12) have been integrated for several stations in Hurricane Baker 1950 and for Hurricane Edna 1954. The paths of these storms are shown in Fig. 2 and typical results are shown in Figs. 3 and 4. These results are interesting in view of the fact that only commonly-accepted values for the stress due to wind and bottom resistance were used. The wind fields were analyzed before the forecast was made.

In order to show that forecast values of the wind will be accurate enough to make good estimates of the storm tide, we increased the wind speeds by an arbitrary 30%. This gives a difference of about 30% in the maximum storm tide, as shown in Figs. 3 and 4. Thus the storm tide forecast is expected to have a linear correlation with the wind forecast.

The agreement of the bathystrophic storm tide with observations at Gulf Shores, Alabama (Fig. 3) is very good. When the pressure term is taken into account, 0.5 foot should be added to the value computed; this gives a computed total of 2.7 feet. The observed maximum value as shown in Fig. 3 is 3.3 feet (including astronomical tide).

At Atlantic City, New Jersey (Fig. 4) the bathystrophic storm tide seems to be a good approximation to the rising and falling tide, but the computed maximum increase over the "astronomical" tide does not occur. There seems to be a disturbance at Atlantic City which is not taken into account by this computation.

First it came to mind that there may have been an error in observing or in recording the observation at Atlantic City, but we were

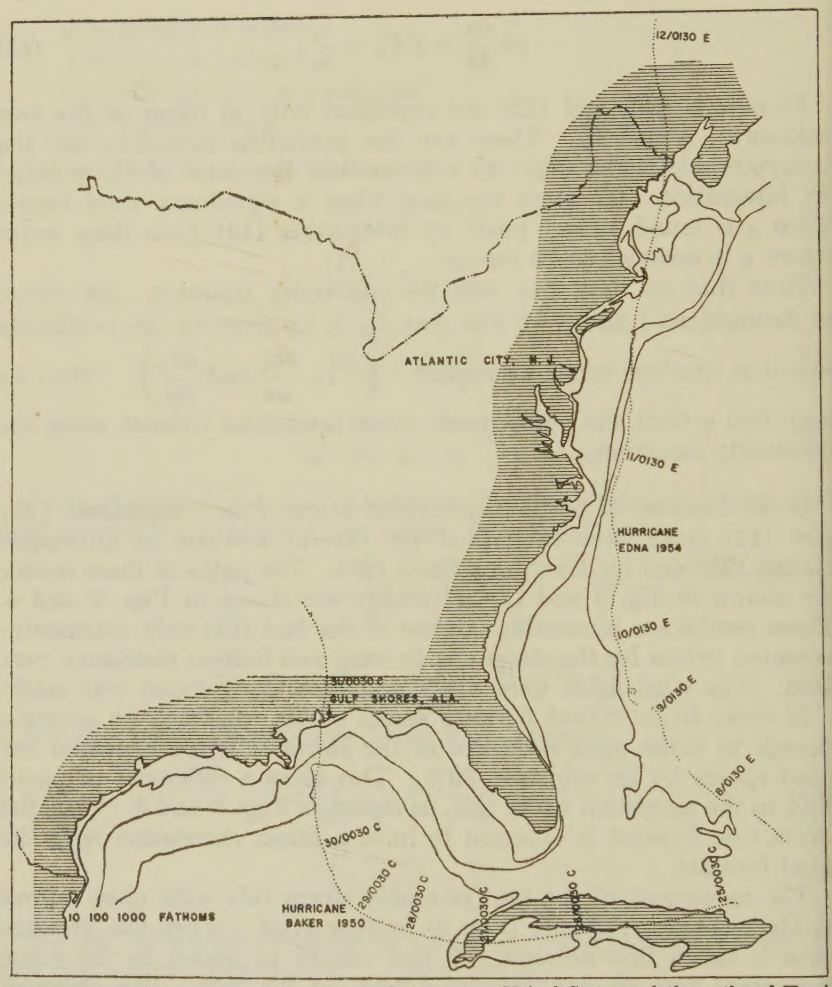

Figure 2. The bathymetry for the shores of the eastern United States and the paths of Hurricane Baker and Hurricane Edna.

assured by Mr. D. Lee Harris of the Weather Bureau that this was not the case. We suggest, then, the possibility that the water flows over a long low barrier or into a bay (the Delaware?) somewhere near Atlantic City and that this action limits the rise of the tide. This causes a "wave" of reduced tide rise to move past Atlantic City.[5]

[5] Many oceanographers will immediately think of edge waves. Munk (1956) has shown that the period of edge waves in this area conforms with the observed period.

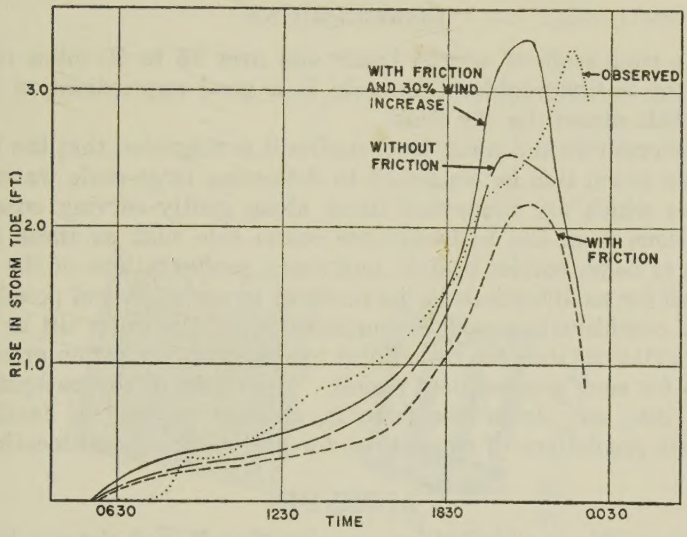

Figure 3. Gulf Shores, Alabama: Various computations of the storm tide during Hurricane Baker. The observed tide includes the astronomical tide at this station.

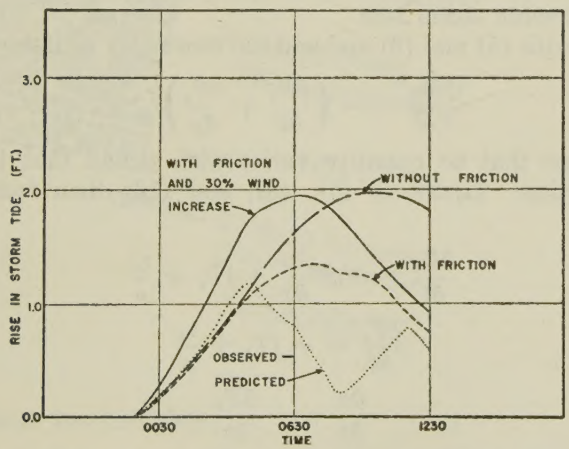

Figure 4. Atlantic City, New Jersey: Various computations of the storm tide during Hurricane Edna. The observed minus predicted tide shows the difference between the actual tide and U. S. Coast and Geodetic Survey computation of the ordinary tide.

A similar occurrence at Sandy Hook indicated a wave moving from Atlantic City northward.

CONCLUSIONS

On a time scale of several hours and over 25 to 50 miles of open coast the bathystrophic storm tide is a good approximation to the storm tide caused by the wind.

For storm tide and water level studies it is suggested that the bathystrophic storm tide be computed to determine large-scale water level changes which are important items along gently-curving coastlines. Deviations from the bathystrophic storm tide such as these due to effects of bays, barrier islands, and sharp configurations of the coastline call for modifications in the problem formulation and possibly for special considerations such as computations of the water tilt in a bay. Such deviations thus can be handled by special computation procedures set up for each geographical region. The study of the bathystrophic storm tide, and deviations from it, appears capable of facilitating accurate predictions of storm tides for particular coastal locations.

APPENDIX

It is possible to show by computation that F_x and changes in η due to variations in F_x remain small while the bathystrophic storm tide builds up. In other words, we can support Assumption 1 with a computation that is (in some ways) more general than the computation of the bathystrophic storm tide.

We start with (5) and (6) and add the continuity equation

$$\frac{\partial \eta}{\partial t} = -\left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y}\right). \tag{13}$$

We assume that no quantity varies with y and that there is no bottom friction. Equations (5), (6), and (13) then reduce to the following:

$$\frac{\partial F_x}{\partial t} = -gh\frac{\partial u}{\partial x} + fT_y + \frac{\tau_x}{\rho} \tag{14}$$

$$\frac{\partial F_y}{\partial t} = -fT_x + \frac{\tau_y}{\rho} \tag{15}$$

$$\frac{\partial u}{\partial t} = -\frac{\partial T_x}{\partial x}. \tag{16}$$

Now we define two new sets of quantities:

$$F_x = F_x'$$

$$\eta = \eta^* + \eta'$$

$$F_y = F_y^* + F_y'.$$